Recoil effects on nucleon electromagnetic form factors

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Abstract

The electromagnetic form factors are computed using eigenstates of linear momentum for the nucleon. The latter is described in the framework of the chiral color-dielectric model, projecting the hedgehog ansatz on eigenstates of angular momentum and isospin. Form factors are well reproduced, with the exception of the magnetic one for the proton, up to $q^2 \simeq 0.5$ GeV^2. The effect of the removal of the spurious center-of-mass contributions shows up mainly in the electric form factor of the proton. A noticeable improvement is obtained with respect to the calculation without linear momentum projection.

1 Introduction

We report on a theoretical calculation of the electromagnetic form-factors of the nucleon in the space-like region, performed in the framework of an effective model — the chiral color-dielectric model (CDM) [1,2] — in which the nucleon is described as a chiral soliton. The model contains quark and meson degrees of freedom and a phenomenological scalar field which is responsible for quark confinement.

In the previous calculations of the form factors in the framework of the chiral soliton models of the nucleon, as the linear sigma model [3], the Nambu-Jona-Lasinio model [4], the Skyrme model [5] and the CDM [6], it was always assumed that the nucleon is at rest before and after the interaction with the virtual photon, the so-called static approximation. In the present work we overcome, at least in part, the technical difficulties associated with the computation of the form factors when the nucleon initial and final states are eigenfunctions of linear momentum, at least non relativistically. Our formalism is a generalization of the one presented in a work by Neuber *et al.* [7,8], where *static* properties of the nucleon have been computed in the framework of the

CDM. In their case therefore it was enough to build eigenstates of the angular momentum having *zero* linear momentum.

The main technical problem in our calculation is due to the non-commutativity of the projectors on linear and on angular momentum. It will be shown that the problem can be solved by taking a suitable average on the direction of the transfered momentum, as it is suggested by the equation $\int \mathrm{d}\hat{\boldsymbol{q}}\,[P_{\boldsymbol{q}}, P_{JM}] = 0$. Morover it will be shown that the Fourier transform of the matrix elements of the electromagnetic current does not depend on the direction of the momentum transfered, if nucleon states are considered. Therefore the integration on $\hat{\boldsymbol{q}}$ does not imply any approximation. The use of the *effective commutativity* of the two projectors simplifies the computation of the doubly projected form factors. These mathematical aspects are applicable in general to any quark-meson chiral soliton model.

The numerical results shown in this paper refer to the electric and the magnetic proton and neutron form factors computed in a particular version of the chiral CDM. This is the so-called "single minimum" version which gives good results in quark matter calculations [9]. The model contains two parameters that we adjust in order to reproduce the average Δ–N mass and the isoscalar nucleon radius. All other results are obtained without any further parameters' fitting.

This paper is organized as follows. In Section 2 the electromagnetic form-factors are defined. In Section 3 we review the chiral color-dielectric model and the way model states representing a nucleon with definite momentum are obtained. Section 4 is devoted to the formalism to compute the electric and the magnetic form factors of the nucleon in the projected hedgehog state. Finally, in Section 5, the results are presented and discussed.

2 The electromagnetic form factors of the nucleon

Let $|N_\alpha(p_\mu)\rangle$ represent a nucleon state of mass M_N, with spin and isospin described by α. The standard definition of the electromagnetic form factors is given by:

$$\langle N_f(p'_\mu)\,|J^\mu_{em}(0)|\,N_i(p_\mu)\rangle = \bar{u}_f(p'_\mu)\left[F_1(q^2_\rho)\gamma^\mu + i\,\frac{F_2(q^2_\rho)}{2M_N}\sigma^{\mu\nu}q_\nu\right]u_i(p_\mu). \quad (1)$$

They only depend on the modulus of the momentum transfer of the virtual photon $q_\mu = p'_\mu - p_\mu$, being real functions of $q_\mu q^\mu = (q^0)^2 - \boldsymbol{q}^2$. We work in the Breit frame where the photon 4-momentum is $q^\mu = (0, \boldsymbol{q})$, i.e. the energy transfer is zero. Our results will be presented as a function of $q = |\boldsymbol{q}|$.

In the Breit frame, Eq. (1) reads explicitly

$$\langle N_f(\frac{\boldsymbol{q}}{2})\,|J^\mu_{em}(0)|\,N_i(-\frac{\boldsymbol{q}}{2})\rangle = \overline{u}_f(\frac{\boldsymbol{q}}{2})\left[F_1(q^2)\gamma^\mu + i\frac{F_2(q^2)}{2M_N}\sigma^{\mu\nu}q_\nu\right]u_i(-\frac{\boldsymbol{q}}{2}). \quad (2)$$

Normalizing the Dirac spinors so that $\overline{u}u = 1$,

$$u(\boldsymbol{p}) = \sqrt{\frac{E+M_N}{2M_N}}\begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+M_N}\end{pmatrix}|\,\chi\rangle, \tag{3}$$

the matrix element (2) can be worked out yielding

$$\begin{aligned}
&\langle N_f(\frac{\boldsymbol{q}}{2})\,|\,J^\mu_{em}\,(0)|\,N_i(-\frac{\boldsymbol{q}}{2})\rangle \\
&\quad = F_1(q^2)\left[\langle\chi_f\,|\chi_i\rangle\delta_{\mu 0} + i\sum_{j=1}^{3}\frac{\langle\chi_f\,|[\boldsymbol{\sigma}\times\boldsymbol{q}]_j|\,\chi_i\rangle}{2M_N}\delta_{\mu j}\right] \\
&\quad - F_2(q^2)\left[\langle\chi_f\,|\chi_i\rangle\frac{q^2}{4M_N^2}\delta_{\mu 0} - i\sum_{j=1}^{3}\frac{\langle\chi_f\,|[\boldsymbol{\sigma}\times\boldsymbol{q}]_j|\,\chi_i\rangle}{2M_N}\delta_{\mu j}\right].
\end{aligned} \tag{4}$$

From $F_1(q^2)$ and $F_2(q^2)$ it is usual to define the so-called Sachs form factors, $G_E(q^2)$ and $G_M(q^2)$, where E and M stand for "electric" and "magnetic" respectively, which are expressed by

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M_N^2}\,F_2(q^2) \tag{5}$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2). \tag{6}$$

Using these definitions and Eq. (4) one obtains explicit formulas for the electric and the magnetic form factors:

$$G_E(q^2)\langle\chi_f\,|\chi_i\rangle = \langle N_f(\frac{\boldsymbol{q}}{2})\,|J^0_{em}(0)|\,N_i(-\frac{\boldsymbol{q}}{2})\rangle \tag{7}$$

$$i\frac{G_M(q^2)}{2M_N}\langle\chi_f\,|\boldsymbol{\sigma}\times\boldsymbol{q}|\,\chi_i\rangle = \langle N_f(\frac{\boldsymbol{q}}{2})\,|\boldsymbol{J}_{em}(0)|\,N_i(-\frac{\boldsymbol{q}}{2})\rangle\,. \tag{8}$$

3 The projected chiral color-dielectric model

In this work we shall use a chiral version of the CDM, whose Lagrangian reads [1,2,7]

$$\mathcal{L} = i\bar{q}\gamma^{\mu}\partial_{\mu}q + \frac{g}{\chi}\bar{q}\left(\sigma_o + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right)q + \frac{1}{2}\left(\partial_{\mu}\chi\right)^2 - V\left(\chi\right)$$
$$+ \frac{1}{2}\left(\partial_{\mu}\sigma_o\right)^2 + \frac{1}{2}\left(\partial_{\mu}\boldsymbol{\pi}\right)^2 - U\left(\sigma_o, \boldsymbol{\pi}\right). \tag{9}$$

This is a model with interacting quarks, chiral mesons σ_o and $\boldsymbol{\pi}$ and also a chiral singlet scalar field χ responsible for confinement. The potential $U\left(\sigma_o, \boldsymbol{\pi}\right)$ in (9) for the chiral mesons is the so-called 'mexican-hat' potential

$$U\left(\sigma_o, \boldsymbol{\pi}\right) = \frac{\lambda^2}{4}\left(\sigma_o^2 + \boldsymbol{\pi}^2 - \nu^2\right)^2 + c\sigma_o + d. \tag{10}$$

The chiral symmetry $SU(2) \times SU(2)$ of $\mathcal{L}$ is explicitly broken by the small term $c\sigma_o$ in (10); the last term in the same expression is a constant fixed in order to have min $U = 0$. The parameters λ and ν are related to the sigma and the pion masses and to the pion decay constant:

$$\lambda^2 = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi}; \qquad \nu^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda^2}. \tag{11}$$

For the χ field potential we consider the quadratic form

$$V(\chi) = \frac{1}{2}M^2\chi^2, \tag{12}$$

where M is the χ mass. It is well known that the chiral CDM allows for soliton solutions in which the quarks are absolutely confined [1,2,7]. In such solutions the χ mean field is a decreasing function of the distance, approaching zero in the limit $r \to \infty$. This generates a raising dynamical mass for the quarks and confines them. In previous works an exhaustive study of the model with a quartic (or 'double minimum') potential was carried out [6,7]. In this work we consider just the quadratic ('single minimum') potential for the confining field. We recently showed [9] that for 'double minimum' potentials and for all sets of parameters fitting nucleon properties, the equation of state for quark matter turns out to be unrealistic. Indeed even at very low density the energy *per* baryon number for quark matter turns out to be smaller than that for nuclear matter. Using instead a quadratic potential for the χ field a realistic equation of state is obtained.

Altogether, the parameters of the model defined by (9) are the pion and sigma masses (fixed at $m_\pi = 0.139$ GeV and m_σ=1.2 GeV), the pion decay constant ($f_\pi = 0.093$ GeV), and g and M, the quark-meson-χ coupling constant and the χ-mass, respectively.

In order to obtain model states representing the nucleon we used the procedure explained in great detail in Ref. [7] which, for the sake of complete-

ness, is sketched here. We consider three valence quarks in the hedgehog state $|\,h\rangle = \frac{1}{\sqrt{2}}(|u\downarrow\rangle - |d\uparrow\rangle)$, all occupying the same lowest positive energy s-orbital and surrounded by clouds of χ, sigmas and pions, described by coherent states ($|\,\Pi\rangle$, for pions; $|\,\Sigma\rangle$, for sigmas; and $|\,\chi\rangle$, for the confining field). The meson mean fields are the expectation values of the field operators in the corresponding coherent states.

We can write the quark single particle states and the meson mean fields as

$$\langle\,\boldsymbol{r}|q\,\rangle = \frac{1}{\sqrt{4\pi}}\begin{pmatrix} u(r) \\ \imath v(r)\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}} \end{pmatrix}|\,h\rangle \tag{13}$$

$$\sigma_o(\boldsymbol{r}) = \langle\Sigma\,|\sigma|\,\Sigma\rangle - f_\pi = \sigma(r) - f_\pi \tag{14}$$

$$\boldsymbol{\pi}(\boldsymbol{r}) = \langle\Pi\,|\boldsymbol{\pi}|\,\Pi\rangle = \frac{\boldsymbol{r}}{r}\phi(r) \tag{15}$$

$$\chi(\boldsymbol{r}) = \langle\chi\,|\chi|\,\chi\rangle = \chi(r)\,, \tag{16}$$

where σ is just the fluctuating part of the σ_o field. Altogether, the hedgehog ansatz reads

$$|\psi_{hh}\,\rangle = \left(C_h^\dagger\right)^3|\,\Pi\rangle\,|\,\Sigma\rangle\,|\,\chi\rangle, \tag{17}$$

where $C_h^\dagger$ creates a particle in the single quark state (13).

Of course, solitons described by the hedgehog $|\,\psi_{hh}\rangle$ cannot represent physical baryons because they are not eigenstates of angular momentum or isospin. In addition, (17) represents a localized object and therefore the translational symmetry of the model hamiltonian is also broken in such states. In particular they contain spurious centre-of-mass components which contribute to the energy and to the other observables. However, a nucleon at rest can be obtained by applying the projector onto linear momentum $\boldsymbol{q} = 0$ together with the projector onto angular momentum-isospin. The linear momentum projector is given by [10]

$$P_{\boldsymbol{q}} = \left(\frac{1}{2\pi}\right)^3\int \mathrm{d}\boldsymbol{a}\,\mathrm{e}^{\imath\boldsymbol{a}\cdot\boldsymbol{q}}U(\boldsymbol{a}), \tag{18}$$

where $U(\boldsymbol{a})$ is the translation operator. It is well known that due to the symmetry of the hedgehog it is enough to perform a single projection (e.g. onto spin), since this automatically projects onto the same value of isospin. The operator which projects out from the hedgehog a state with angular momentum J and isospin $T = J$ is

$$P^J_{MM_T} = (-1)^{J+M_T}\frac{2J+1}{8\pi^2}\int \mathrm{d}^3\Omega\,\mathcal{D}^{J^*}_{M,-M_T}(\Omega)R(\Omega), \tag{19}$$

where $\Omega = (\alpha, \beta, \gamma)$ are the three Euler angles, $\mathcal{D}^J_{M,K}(\Omega)$ are the Wigner functions and $R(\Omega)$ is the rotation operator. In the following we will consider $M_T = -M$ and use the shorthand notation $P_{JM} \equiv P^J_{M\,-M}$.

The radial functions in (13)-(16) are determined using an approximate variation-after-projection method firstly suggested by Leech and Birse [11]. They are computed by minimizing the expectation value of the (normal-ordered) model hamiltonian in the model baryon state with quantum numbers $J = T = \frac{1}{2}$ and linear momentum zero:

$$P_{\boldsymbol{q}=0} P_{JM} \, |\Psi_{hh}\rangle = |J, T, M, \boldsymbol{q} = 0\,\rangle \,. \tag{20}$$

In the model there are two parameters, g and M, yet to be fixed. However, because of the smoothness of the χ-field in a typical soliton solution and of the relative weakness of the chiral meson clouds, the relevant parameter turns out to be $G = \sqrt{gM}$. In the quark matter sector this is indeed the only free parameter of the model [12]. If G is fixed to reproduce the isoscalar radius of the nucleon one obtains $G = 0.2$ GeV. For this parameter the nucleon-delta average mass is around 1.13 GeV (experimental value 1.085 GeV). It is interesting to observe that if g and M are changed, keeping G fixed, the static properties of the nucleon are essentially unchanged. For example, for $G = 0.2$ GeV, changing the mass of the χ field in the range 0.8–2.0 GeV, affects the results by less than 1%.

In the present version of the CDM, the nucleon-delta mass splitting results only from the quark-pion interaction. Due to the weakness of the pionic field, the nucleon delta mass splitting obtained is too small. The experimental value of the splitting could be recovered if, in addition, a color-magnetic interaction (like in the MIT bag model or in the cloudy bag model) was considered [13]. We will come back to this point in the conclusions.

4 Electromagnetic form factors in the projected hedgehog

In order to compute the electromagnetic nucleon form factors one has to evaluate matrix elements of the electromagnetic current operator. In the CDM the latter is given by

$$J^\mu_{em}(x) =: \sum_{a=1}^{3} \bar{q}_a(x) \gamma^\mu \left(\frac{1}{6} + \frac{\tau_3}{2} \right) q_a(x) + [\boldsymbol{\pi}(x) \wedge \partial^\mu \boldsymbol{\pi}(x)]_3 \; : \,. \tag{21}$$

As mentioned in the Introduction, in the past the matrix elements of this operator have been computed in the *static* approximation (the nucleon is assumed

to be at rest *before* and *after* the interaction with the photon). In the present work we go beyond this approximation, since we compute the matrix elements using momentum eigenstates. In principle these should be obtained by boosting [14] the nucleon zero momentum eigenstate (20). However, the technical difficulties associated with boosting are prohibitive. We approximate this operation by a Peierls-Yoccoz projection, i.e. we consider our model state with momentum $\boldsymbol{q}$ to be given by

$$|N(q)\rangle \rightarrow \sqrt{(2\pi)^3\delta^3(0)}\sqrt{\frac{E}{M_N}}\frac{P_{\boldsymbol{q}}\,|\Psi_{JM}\rangle}{\sqrt{\langle P_{\boldsymbol{q}}\,\Psi_{JM}\,|P_{\boldsymbol{q}}\,\Psi_{JM}\rangle}}\,, \tag{22}$$

where the square roots are just normalization factors and

$$|\Psi_{JM}\rangle = P_{JM}\,|\Psi_{hh}\rangle. \tag{23}$$

The approximation involved in assuming projected instead of boosted states is valid for small $\boldsymbol{q}$.

Before presenting the formalism to compute the form factors as matrix elements of the electromagnetic current (21) taken between nucleon states, let us recall that

$$\int \mathrm{d}\boldsymbol{z}\ [F(\boldsymbol{z})U(\boldsymbol{z}), R(\Omega)] = 0, \tag{24}$$

if $F(\boldsymbol{z})$ is a scalar function of $\boldsymbol{z}$ [15]. This can be seen writing explicitly the commutator, rotating the argument of F and U and redefining $\boldsymbol{z}' = \mathcal{R}(\Omega)\boldsymbol{z}$. The previous commutation relation will be very useful in the following.

Another important point is that, due to the symmetry of the hedgehog, rotations of this state in spin or isospin space are equivalent. Therefore, as it was already pointed out in the previous Section, projecting the hedgehog onto spin J implies a simultaneous projection onto isospin $T = J$, and the two projections are equivalent. Hence, the following relations hold:

$$P_{\boldsymbol{q}}P_{JM}|\Psi_{hh}>= P_{\boldsymbol{q}}P_{TM}|\Psi_{hh}>= P_{TM}P_{\boldsymbol{q}}|\Psi_{hh}>\neq P_{JM}P_{\boldsymbol{q}}|\Psi_{hh}>, \tag{25}$$

where the commutation between operators working in isospin space and operators working in ordinary space has been used. The projector P_{TM} is defined similarly to the projector P_{JM} (Eq. (19)), but with the rotation operator R acting in isospin space, replacing the rotation operator in spin space. We shall exploit relations (25) in the evaluation of the magnetic form factors.

4.1 Electric form factor

From the definition of the electric form factor (7) and using the correspondence (22), the electric form-factor is given by

$$G_E(q^2) = \frac{E}{M_N} \frac{\int \mathrm{d}\boldsymbol{x}\, \langle P_{\frac{\boldsymbol{q}}{2}} \Psi_{JM} | J^0_{em}(0) | P_{-\frac{\boldsymbol{q}}{2}} \Psi_{JM} \rangle}{\langle P_{\frac{\boldsymbol{q}}{2}} \Psi_{JM} | P_{\frac{\boldsymbol{q}}{2}} \Psi_{JM} \rangle}\,, \tag{26}$$

and using the explicit form of the linear momentum projector (18) one obtains

$$G_E(q^2) = \frac{E}{M_N} \frac{\int \mathrm{d}\boldsymbol{x}\, \mathrm{d}\boldsymbol{b}\, \mathrm{d}\boldsymbol{b}'\, e^{-i(\boldsymbol{b}+\boldsymbol{b}')\cdot\frac{\boldsymbol{q}}{2}} \langle U(\boldsymbol{b}')\Psi_{JM} | J^0_{em}(0) | U(\boldsymbol{b})\Psi_{JM} \rangle}{\int \mathrm{d}\boldsymbol{b}\, \mathrm{d}\boldsymbol{b}'\, e^{-i(\boldsymbol{b}-\boldsymbol{b}')\cdot\frac{\boldsymbol{q}}{2}} \langle U(\boldsymbol{b}')\Psi_{JM} | U(\boldsymbol{b})\Psi_{JM} \rangle}\,. \tag{27}$$

In principle the form factors should be functions of q^2 only. However, due to the approximate treatment of the center-of-mass motion this is in general no longer guaranteed and there is a spurious dependence on the angle between $\mathbf{q}$ and the quantization direction. However, if $J = 1/2$ states are considered, it is possible to show that the form factor is indeed a function of q^2 only. In fact, due to parity, the Fourier transform of the matrix element of the current has to be a function of $(\mathbf{q}\cdot\mathbf{J})^{\mathbf{2}}$ which, for $J = 1/2$, is proportional to q^2. We can therefore integrate the direction $\hat{\boldsymbol{q}}$, both in the current matrix element and in the normalization factor at the denominator. After the expansion of the exponentials in spherical waves, only the $\ell = 0$ wave contributes and one gets

$$G_E(q^2) = \frac{E}{M_N} \frac{\int \mathrm{d}\boldsymbol{x}\, \mathrm{d}\boldsymbol{b}\, \mathrm{d}\boldsymbol{b}'\, j_0\left(|\frac{\boldsymbol{b}+\boldsymbol{b}'}{2}|q\right) \langle U(\boldsymbol{b}')\Psi_{JM} | J^0_{em}(0) | U(\boldsymbol{b})\Psi_{JM} \rangle}{\int \mathrm{d}\boldsymbol{b}\, \mathrm{d}\boldsymbol{b}'\, j_0\left(|\frac{\boldsymbol{b}-\boldsymbol{b}'}{2}|q\right) \langle U(\boldsymbol{b}')\Psi_{JM} | U(\boldsymbol{b})\Psi_{JM} \rangle}\,. \tag{28}$$

The integration on $\hat{\boldsymbol{q}}$ allows for further simplifications. In fact, we can now prove that the translation operator $U(\boldsymbol{b})$ and the rotation operator $R(\Omega)$, which enters the projector on angular momentum [Eq. (19)], can be exchanged in the previous formula, although they don't commute. Let us, first of all, note that

$$U^\dagger(\boldsymbol{b}') J^0_{em}(0) U(\boldsymbol{b}) = J^0_{em}(\boldsymbol{b}') U(\boldsymbol{b}-\boldsymbol{b}') \tag{29}$$

and expand the spherical Bessel functions j_0 in power series of $|\boldsymbol{b} \pm \boldsymbol{b}'|q/2$. Notice that the integration on the direction of the momentum transfer has eliminated the dependence of the integrand in Eq. (28) on the angle between $\boldsymbol{q}$ and $\boldsymbol{b} \pm \boldsymbol{b}'$. After the introduction of the new variables

$$\boldsymbol{z} = \boldsymbol{b} - \boldsymbol{b}' \quad , \quad \boldsymbol{y} = \boldsymbol{b}' \tag{30}$$

we can define the scalar function

$$F_{s,m}(\boldsymbol{z}) = \int \mathrm{d}\boldsymbol{y} \;\; y^{2m+s} \, (\cos\theta_{zy})^s \, J^0_{em}(\boldsymbol{y}) \tag{31}$$

where θ_{zy} is the angle between the directions $\boldsymbol{z}$ and $\boldsymbol{y}$. The l.h.s. of Eq. (28) can be written as a sum of terms of the form:

$$\int \mathrm{d}\boldsymbol{z} \, \langle \Psi_{hh} | P_{JM} F_{s,m}(\boldsymbol{z}) U(\boldsymbol{z}) P_{JM} | \Psi_{hh} \rangle \, z^{2k+s} \, . \tag{32}$$

We can now apply Eq. (24) to each of the previous terms so that one of the two projectors on angular momentum in Eq. (28) can be eliminated both in the numerator and in the denominator. The electric form factor can finally be written as

$$\begin{aligned} G_E(q^2) = & \frac{1}{\mathcal{N}} \frac{E}{M_N} \int \mathrm{d}\boldsymbol{a} \int \mathrm{d}\boldsymbol{r} \int \mathrm{d}^3\Omega \; j_0(qr) \mathcal{D}^{J^*}_{MM}(\Omega) \\ & \times < U(-\frac{\boldsymbol{a}}{2}) \Psi_{hh} | J^0_{em}(\boldsymbol{r}) | U(\frac{\boldsymbol{a}}{2}) R(\Omega) \Psi_{hh} > \end{aligned} \tag{33}$$

where the new variables $\boldsymbol{a}$ and $\boldsymbol{r}$ are related to the previous $\boldsymbol{b}$ and $\boldsymbol{b}'$ through

$$\boldsymbol{b}' = \boldsymbol{r} - \boldsymbol{a}/2 \tag{34}$$

$$\boldsymbol{b} = \boldsymbol{r} + \boldsymbol{a}/2 \tag{35}$$

and the normalization factor reads

$$\mathcal{N} = \int \mathrm{d}\boldsymbol{a} \int \mathrm{d}^3\Omega \, j_0(\frac{qa}{2}) \, \mathcal{D}^{J^*}_{MM}(\Omega) < \Psi_{hh} | U(\boldsymbol{a}) R(\Omega) | \Psi_{hh} > . \tag{36}$$

The numerator of Eq.(33) is the sum of three pieces [see Eq. (21)]: the isoscalar quark, the isovector quark and the isovector pion contributions.

4.2 Magnetic form factor

From the definition of the magnetic form factor (8) and taking again the correspondence (22) we can write

$$i \, \frac{G_M(q^2)}{2M_N} (\boldsymbol{\alpha} \times \boldsymbol{q}) = \frac{E}{M_N} \frac{\int \mathrm{d}\boldsymbol{x} \, \langle P_{\frac{\boldsymbol{q}}{2}} \Psi_{JM} | \boldsymbol{J}_{em}(0) | P_{-\frac{\boldsymbol{q}}{2}} \Psi_{JM} \rangle}{\langle P_{\frac{\boldsymbol{q}}{2}} \Psi_{JM} | P_{\frac{\boldsymbol{q}}{2}} \Psi_{JM} \rangle} \tag{37}$$

where we have introduced $\boldsymbol{\alpha} = \langle \boldsymbol{\sigma} \rangle$.

In the case of the magnetic form factors, where the space components of the electro-magnetic current appear, it is not possible to define a *scalar* function as we did in Eq. (31). The projection operators will therefore always rotate in a non-trivial way the current. To simplify the expression of the magnetic form factor we will instead make use of the relations (25). The current matrix element can therefore be rewritten as

$$\begin{aligned}\langle\Psi_{hh}|P_{JM}P_{\frac{\boldsymbol{q}}{2}}\,\boldsymbol{J}_{em}\,(0)P_{-\frac{\boldsymbol{q}}{2}}P_{JM}|\Psi_{hh}\rangle &= \langle\Psi_{hh}|P_{TM}P_{\frac{\boldsymbol{q}}{2}}\boldsymbol{J}_{em}(0)P_{-\frac{\boldsymbol{q}}{2}}P_{TM}|\Psi_{hh}\rangle \\ &= \langle\Psi_{hh}|P_{\frac{\boldsymbol{q}}{2}}P_{TM}\boldsymbol{J}_{em}(0)P_{TM}P_{-\frac{\boldsymbol{q}}{2}}|\Psi_{hh}\rangle\,. \end{aligned}\tag{38}$$

The current contains an isoscalar and an isovector piece. The first one commutes with the current, while the isovector one transforms as [16]:

$$P^T_{MM}\,\boldsymbol{J}^{iv}_{em\,,0}\,P^T_{MM} = \sum_{Q=-1}^{+1} C_Q(T,M)\boldsymbol{J}^{iv}_{em,\,Q}P^T_{M-Q,M}\tag{39}$$

where $\boldsymbol{J}^{iv}_{em\,,Q}$ stands for the spherical isospin component Q of the isovector part of the vector electromagnetic current and

$$C_Q(T,M) = \langle 10;\,TM|TM\rangle\langle 1Q;\,T\,M{-}Q|TM\rangle.\tag{40}$$

To extract the magnetic form factor out of Eq. (37) we multiply both terms of the equation by $\boldsymbol{\alpha}\times\boldsymbol{q}$ and integrate over $\hat{\boldsymbol{q}}$. As for the electric form factors, this integration is trivial because the Fourier transform of the matrix element depends only on q^2.

After a straightforward algebraic derivation on obtains the isoscalar (*is*) part of the magnetic form factor:

$$\begin{aligned}\frac{G^{is}_M(q^2)}{2M_N} = &\frac{1}{\mathcal{N}}\frac{E}{M_N}\int \mathrm{d}\boldsymbol{a}\int \mathrm{d}\boldsymbol{r}\int \mathrm{d}^3\Omega\,\frac{3j_1(qr)}{2qr}\mathcal{D}^{J^*}_{MM}(\Omega) \\ &\times\sum_{jk}\epsilon_{3jk}\,\langle U(-\frac{\mathbf{a}}{2})\Psi_{hh}|\,r_j[\boldsymbol{J}^{is}_{em}(\boldsymbol{r})]_k\,|U(\frac{\mathbf{a}}{2})R(\Omega)\Psi_{hh}\rangle,\end{aligned}\tag{41}$$

where $[\boldsymbol{J}^{is}_{em}]_k$ stands for the cartesian k component of the vector electromagnetic current (isoscalar part) and the normalization factor $\mathcal{N}$ is given by Eq. (36), as before.

For the isovector (*iv*) part of the magnetic form factor we get

$$
\frac{G_M^{iv}(q^2)}{2M_N} = \frac{1}{\mathcal{N}} \frac{E}{M_N} \sum_Q C_Q \int \mathrm{d}\boldsymbol{a} \int \mathrm{d}\boldsymbol{r} \int \mathrm{d}^3\Omega \, \frac{3j_1(qr)}{2qr} \mathcal{D}^{J\,*}_{M-Q,M}(\Omega)
$$

$$
\times \sum_{jk} \epsilon_{3jk} \, \langle U(-\frac{\boldsymbol{a}}{2})\Psi_{hh} | \, r_j [\boldsymbol{J}^{iv}_{em,Q}(\boldsymbol{r})]_k \, | U(\frac{\boldsymbol{a}}{2}) R(\Omega) \Psi_{hh} \rangle \, . \qquad (42)
$$

5 Results and discussion

The nucleon electric and magnetic form factors are presented in Figs. 1-4. The experimental data shown were taken from Refs. [17–19]. We present our results in the space like region $0 \leq q^2 \leq 0.5$ GeV^2.

All the results in Figures 1–4 were obtained using $G = 0.2$ GeV (actually $g = 0.024$ GeV and $M = 1.7$ GeV but, as mentioned before, the results basically depend on the combination $G = \sqrt{gM}$). No parameter was fitted to reproduce the experimental form factors.

For the sake of comparison, in Figs. 1-4 we also present the results for the form factors computed in the static approximation, i.e. without performing the linear momentum projection. This is the traditional approximation considered in previous calculations of the form factors in the framework of soliton models (see Ref. [3] for the linear sigma model and Ref. [6] for the double hump version of the chiral CDM).

As it can be seen from Figs. 1-2, the electric form factors are rather satisfactory. One has to take into account large incertitudes in the experimental analysis of the electric form factor of the neutron, which is obtained from scattering on the deuteron and depends hence on the wave function of the latter. The effect of the projection on linear momentum is particularly relevant in the proton electric form factor.

The magnetic form factors are less satisfactory. This is probably due to the weakness of the spin-spin interaction obtainable in this model, at least working within the projected mean-field approximation.

As it appears from the figures, all the computed form factors underestimate the data for large q^2. It can be interesting to note that, also studying structure functions, one sees that in the region of large $x = Q^2/2M\nu$, where the momentum carried by the quarks is large, the computed quantities underestimate the data [20]. These problems are probably due to the approximate treatment of translational invariance and are therefore not so much related to the specific model used in this paper.

The chiral CDM has now been used to compute many different quantities.

We can try to summarize the results. The chiral CDM gives good results if problems not involving the spin are considered. This can be seen also from the computation of the unpolarized structure functions [20]. Also the study of the transition from nuclear to quark matter within this model seems very promising, suggesting a smooth transition between the two phases [9] and giving interesting results for neutron stars [21].

On the other hand, since the model does not contain enough tensor force, it provides poor results for observables which involve the spin. The magnetic form factors are therefore not totally satisfactory and the polarized structure functions overestimate the data [20], indicating that most of the spin is carried by the quarks because the pion is very weak. It is not yet clear whether the weakness of the pionic field is intrinsic to the model and therefore other degrees of freedom have to be considered, or stems from the approximations used to solve the field equations. Concerning this second possibility, there are indications that a large Δ-N mass splitting could be obtained using the same ingredients considered here but allowing the scalar and the vector diquarks to have different radii [22].

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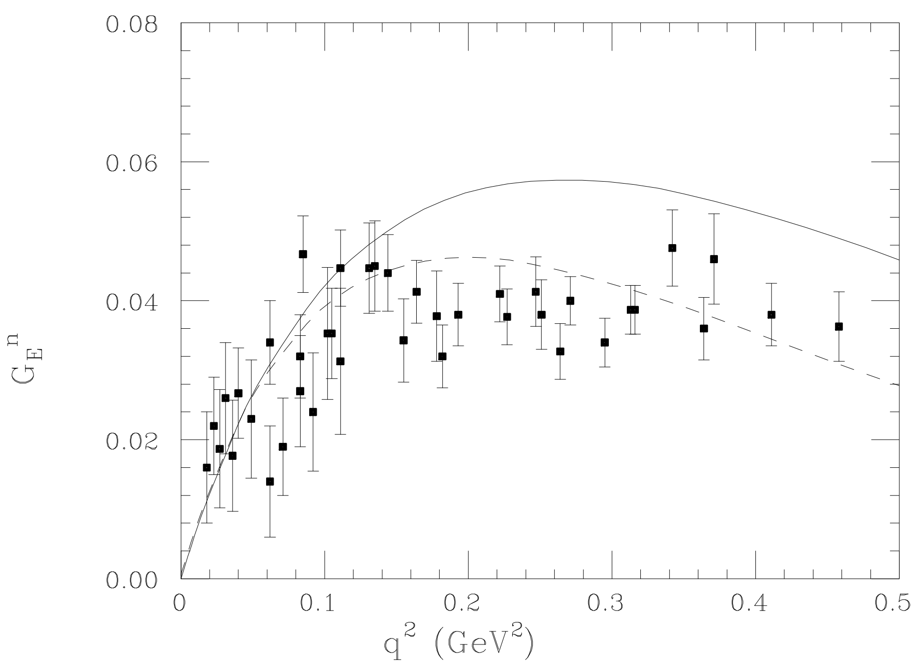

Fig. 1 Electric form factor of the neutron as a function of momentum transfer in the space like region. The dashed line corresponds to the static approximation. The full curve corresponds to the full calculation, i.e. combined linear and angular momentum projections.

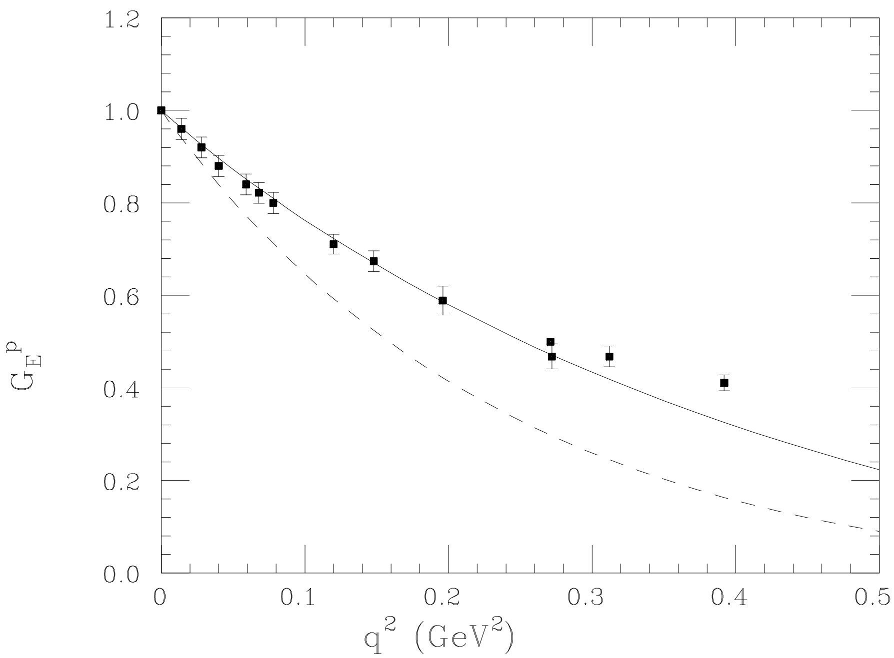

Fig. 2 Electric form factor of the proton. Dashed and full curves as in Fig. 1.

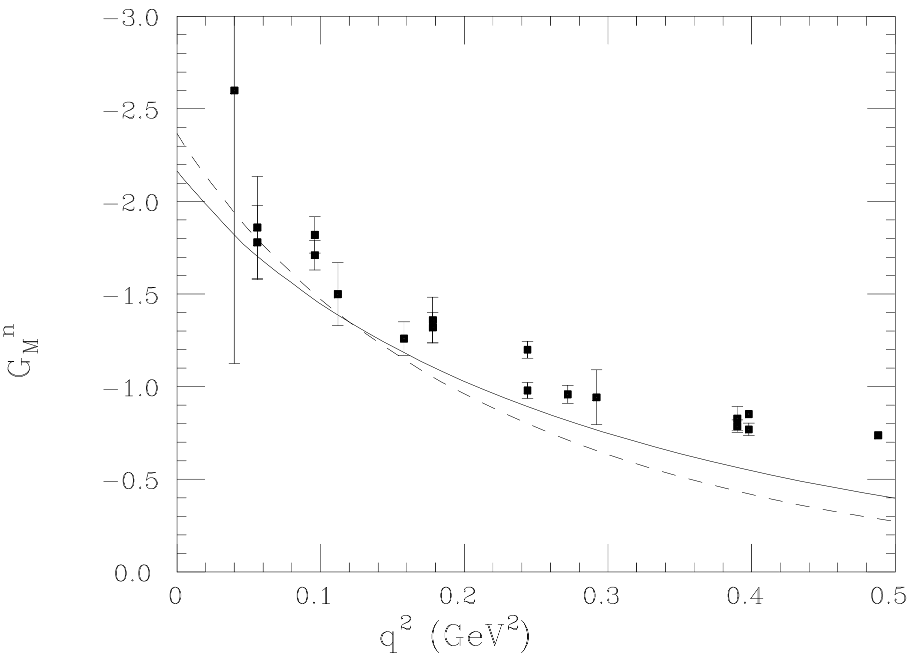

Fig. 3 Magnetic form factor of the neutron. Dashed and full curves as in Fig 1.

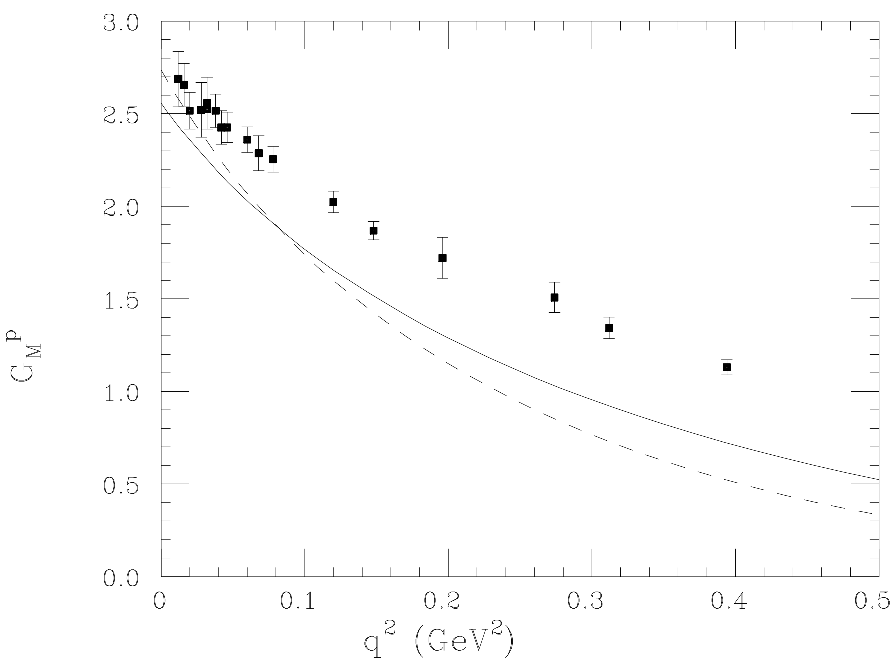

Fig. 4 Magnetic form factor of the proton. Dashed and full curves as in Fig 1.